I. GENERAL REMARKS

In this supplement we describe our event driven algorithm along with the collision rules relevant for motion of the dimer in the drift (D) mode.

For the convenience of reading we recall here the general approach. Our dimer is made of two heavy spheres of radius $R$ connected by a light (weightless) rod. The distance between the centers of spheres is $L$. The dynamics of the dimer is governed by the Newton equation written for the center of mass (CM) velocity $v = (u, 0, v)$ and the angular velocity $\omega = (0, \omega, 0)$

$$m \ddot{v} = \sum_c c^c + mg, \quad I \ddot{\omega} = \sum_c r_c \times F^c$$

where $m, I = m(L^2/4 + 2/5R^2)$ are the dimer mass and moment of inertia, $F^c = (F_x^c, 0, F_z^c)$ is the contact forces, and $r_c = (-X_c, 0, -Z_c)$ is radius-vector from CM to a contact point (CP) [see also Fig. 1(b) in the text]. The sum over $c \in \{l, r\}$ indicates that either left or right or both spherical ends can be in contact with the plate during collisions. The CP velocity components in the plate frame of reference are given by $V^c = V_c + \omega X_c - V_{pl}$, where $V_{pl}$ is the plate velocity. In a sliding contact, $F^c = -\mu_F c^c$, where $\mu_F$ is the friction coefficient for short collisions and $j = s \mu_{sF}$, and in the sticking phases, we require $V_c = 0$. Transitions from stick to slip occur when $|F^c| = \mu_s F^c$ with the static friction coefficient $\mu_s$. In rolling phases, we introduce different dynamic and static friction coefficients $\mu$ and $\mu_s$, respectively.

We assume instantaneous collisions, therefore no macroscopic change in the tilt angle $\phi$ (see Fig. 1(b) in the text) and the position of the dimer occurs during collision and the gravity force $mg$ can be neglected. However it has to be taken into account for description of the rolling regimes.

For short-time collisions, we integrate eq. (1) with initial conditions $(u^b, b^b, \omega^b)$ and the kinematic condition $V^a = -\epsilon v^a$ to obtain after-collision CM velocities $(u^a, v^a, \omega^a)$. The indices $b, a$ correspond to before and after collision, and $\epsilon$ is the kinematic restitution coefficient.

First, we consider “single” collisions when dimer has one contact point with the plate during an impact. Single collisions may be of three different types, depending on the sign of the horizontal component of the contact velocity before and after impact,

1. CL: continuous slide $U^b U^a > 0$
2. SS: slip-stick $U^a = 0$
3. SR: slip reversal $U^b U^a < 0$

In addition to three types of isolated single-ball collisions, for $\Gamma < 1$ it is possible that one of the balls maintains a prolonged contact with the plate while the other ball is off the plate, and so two different rolling phases are possible,

1. RO: rolling without slip $U = 0$
2. RS: rolling with slip $U \neq 0$

Next we present formulas for three cases of “double” collision ensuing after the rolling phase, when dimer has two contact points,

1. DS: double slide $U^b U^a > 0$ and $V^b = 0$ or $V^b = 0$
2. DSS: double slip-stick $U^a = 0$ and $V^b = 0$ or $V^b = 0$
3. DSR: double slip reversal $U^b U^a = 0$ and $V^b = 0$ or $V^b = 0$

Finally, we describe how all pieces are combined into event driven simulations.
II. SINGLE COLLISIONS

In this section we write down the formulas for an isolated collision between a dimer and a plate. These formulas are deduced from our previous paper (with some change of the notations) [2]. The general mappings which relate CM velocities immediately after collision to the CM velocities right before collision may be written as,

\[ u^a = F_u(u^b, v^b, \omega^b, \phi) \]
\[ v^a = F_v(u^b, v^b, \omega^b, \phi) \]
\[ \omega^a = F_\omega(u^b, v^b, \omega^b, \phi) \]

A. Slide

The continuous sliding condition reads

\[ j \left[ u^b - \omega^b Z + \frac{XZ + j\mu(K^2 + Z^2)}{K^2 + X^2 + j\mu XZ} (v^b + \omega^b X - V_{pl})(1 + \epsilon) \right] > 0 \] (5)

Here \( j = \text{sgn}(u^b - \omega^b Z) \) is the direction of the horizontal projection of the contact point velocity in the beginning of the contact, and the expression in square brackets is the horizontal contact velocity in the end of the contact. If this condition is satisfied, the colliding ball slides throughout the collision without changing direction. The mapping for the CM velocities at the end of the collision reads:

\[ F_u(u, v, \omega, \phi) = u + \frac{j\mu(1 + \epsilon)K^2(v - V_{pl} + \omega X)}{K^2 + X^2 + j\mu XZ}, \] (6)
\[ F_v(u, v, \omega, \phi) = v - \frac{(v - V_{pl} + \omega X)(1 + \epsilon)K^2}{K^2 + X^2 + j\mu XZ}, \] (7)
\[ F_\omega(u, v, \omega, \phi) = \omega - \frac{(v - V_{pl} + \omega X)(1 + \epsilon)(X + j\mu Z)}{K^2 + X^2 + j\mu XZ}, \] (8)

B. Slip-stick

If condition (5) is violated, the horizontal contact velocity turn zero some time during the contact (while the normal force is still non-zero). If additionally

\[ j[\mu_s(K^2 + Z^2) - XZ] > 0, \] (9)

(where \( \mu_s \) is the static friction coefficient) the contact sticks \((U^a = 0)\) and we get

\[ F_u(u, v, \omega, \phi) = \frac{Z^2(u - \omega Z) - (1 + \epsilon)XZ(v - V_{pl} + \omega X)}{K^2 + X^2 + Z^2} + \omega Z \] (10)
\[ F_v(u, v, \omega, \phi) = \frac{[X^2 - \epsilon(K^2 + Z^2)](v - V_{pl} + \omega X) - XZ(u - \omega Z) + V_{pl} - \omega X}{K^2 + X^2 + Z^2} \] (11)
\[ F_\omega(u, v, \omega, \phi) = \omega + \frac{Z(u - \omega Z) - (1 + \epsilon)X(v - V_{pl} + \omega X)}{K^2 + X^2 + Z^2} \] (12)

C. Slip reversal

If condition (5) is violated and

\[ j[\mu_s(K^2 + Z^2) - XZ] < 0, \] (13)
the contact slides back after stopping and we obtain

\[
\begin{align*}
F_u(u, v, \omega, \phi) &= u - \frac{2j\mu K^2(K^2 + X^2)(u - \omega Z)}{(j\mu(K^2 + Z^2) + XZ)(K^2 + X^2 - j\mu ZX)} - \frac{j\mu K^2(1 + \epsilon)(v - V_{pl} + \omega X)}{K^2 + X^2 - j\mu ZX} \\
F_v(u, v, \omega, \phi) &= v - \frac{2j\mu K^2 ZX(u - \omega Z)}{(j\mu(K^2 + Z^2) + XZ)(K^2 + X^2 - j\mu ZX)} - \frac{(1 + \epsilon)(j\mu Z - X)(v - V_{pl} + \omega X)}{K^2 + X^2 - j\mu ZX} \\
F_\omega(u, v, \omega, \phi) &= \omega + \frac{2j\mu K^2 Z(u - \omega Z)}{(j\mu(K^2 + Z^2) + XZ)(K^2 + X^2 - j\mu ZX)} + \frac{(1 + \epsilon)(j\mu Z - X)(v - V_{pl} + \omega X)}{K^2 + X^2 - j\mu ZX}
\end{align*}
\]

III. ROLLING PHASE

In this section we consider the regime in which one end of the dimer lies on the plate and the other bounces. In this case the gravity force has to be taken into account together with the normal reaction force and the friction force at the contact point. The Newton equations for the CM (1) for this case read

\[
\begin{align*}
m\dot{u} &= F_x^c \\
\dot{v} &= F_z^c - m(g + \dot{V}_{pl}) \\
L\dot{\omega} &= X F_x^c - Z F_z^c
\end{align*}
\]

The equation for the contact point velocity \( V_c = \{U_c, 0, V_c\} \) read

\[
m\dot{V}_c = F + mI^{-1}(r_c \times F_c) \times r_c + m\omega \times r_c - m\dot{V}_{pl},
\]

or in projections

\[
\begin{align*}
m\dot{U}_c &= -\frac{XZ}{K^2} F_x^c + \frac{K^2 + Z^2}{K^2} F_z^c - m\omega \dot{Z} \\
m\dot{V}_c &= \frac{K^2 + X^2}{K^2} F_x^c - \frac{XZ}{K^2} F_z^c - m(g + \dot{V}_{pl}) + m\omega \dot{X}
\end{align*}
\]

A. Rolling without slip

For rolling without slip, \( U_c = 0 \) and \( V_c = 0 \), so Eqs.(21),(22) can be solved with respect to \( F_{x,z}^c \),

\[
\begin{align*}
F_x^c &= m \frac{XZ(g + \dot{V}_{pl} - \omega \dot{X}) + \omega(K^2 + X^2)\dot{Z}}{K^2 + X^2 + Z^2} \\
F_z^c &= m \frac{(K^2 + Z^2)(g + \dot{V}_{pl} - \omega \dot{X}) + \omega XZ\dot{Z}}{K^2 + X^2 + Z^2}
\end{align*}
\]

These forces can be used in Eqs.(17)-(19) to determine the CM velocities. Expressing \( \omega = \dot{\phi}, X = L \sin \phi/2, Z = L \cos \phi/2 + R \) for a dimer with bouncing right end, we re-write Eq.(19) as a close equation for the tilt angle \( \phi \),

\[
(7R^2/5 + L^2/2 + LR \cos \phi)\ddot{\phi} - LR \sin \phi \dot{\phi}^2/2 = (g + \dot{V})L \sin \phi/2
\]

The rolling phase without slip ends when either of the following occurs,

(i) \( |F_x^c/F_z^c| = \mu_s \), transition to rolling with slip

(ii) \( F_x^c = 0 \), dimer leaves the plate

(iii) \( \phi = \pi/2 \), the other end collides with the plate.
B. Rolling with slip

For rolling with slip, the frictional force \( F_z^c = -j\mu F_z^c \) where \( j \) is the direction of the slip, \( j = \text{sgn}U_c \). In this case the equations of motion read,

\[
\begin{align*}
\dot{m}u &= -j\mu F_z^c \\
\dot{m}v &= F_z^c - mg \\
I\dot{\omega} &= (X + j\mu Z)F_z^c
\end{align*}
\]

We can find \( F_z^c \) from Eq.(27) and substitute it in Eq.(28),

\[
I\dot{\omega} = (X + j\mu Z)(\dot{v} + g)
\]

Using \( v = \dot{Z} + V_{pl}, \omega = \dot{\phi}, X = L\sin\phi/2, Z = L\cos\phi/2 + R, K^2 = L^2/4 + 2R^2/5 \), we re-write Eqs.(29)

\[
\frac{L}{2}\text{sgn}(\cos\phi)[\dot{\phi}\sin\phi + \cos\phi\dot{\phi}]^2 = -\frac{(L^2/2 + 4R^2/5)\ddot{\phi}}{L\sin\phi + j\mu[L\cos\phi + 2R]} + g + \dot{V}_{pl}
\]

The normal force is given by

\[
F_z^c = \frac{(L^2/2 + 4R^2/5)\ddot{\phi}}{L\sin\phi + j\mu[L\cos\phi + 2R]}
\]

The horizontal slip velocity \( U = u - \omega Z \) is found from the equation

\[
\dot{U} = -\frac{j\mu(L^2/2 + 4R^2/5)\ddot{\phi}}{L\sin\phi + j\mu[L\cos\phi + 2R]} - (\frac{L}{2}\cos\phi + R)\ddot{\phi} + \frac{L}{2}\sin\phi\text{sgn}(\cos\phi)\dot{\phi}^2
\]

The rolling phase with slip ends when either of the following occurs.

(i) \( U = 0 \) and \( |F_z^c/F_z^c| < \mu_s \), transition to rolling without slip

(ii) \( U = 0 \) and \( |F_z^c/F_z^c| > \mu_s \), transition to rolling with reversed slip direction

(iii) \( F_z^c = 0 \), dimer leaves the plate

(iv) \( \phi = \pm\pi/2 \) the other end collides with the plate.

Note that \( F_z^c \) and \( F_z^c \) in conditions (i) and (ii) have to be computed using formulas (23),(24).

IV. DOUBLE COLLISIONS

Double collision occurs when one spherical end rolls (with or without slip) on the plate when the other end collides with the plate. At the time of double collision \( \phi = (2n + 1)\pi/2, X = L/2\text{sgn}(\sin\phi), \) and \( Z = R \). For definiteness, we assume that the left end of the dimer rolls on the plate with the contact slip velocity \( U_b^l \), and the right tip collides with the plate with the relative vertical velocity \( V_{pb}^h(< 0) \), so \( X = L/2 \). We impose the kinematic restitution condition for the right tip of the dimer, so its vertical contact velocity at the end of the collision is \( -eV_{pb}^h \).

As usual, during collision we can neglect the gravity and non-inertial forces and only consider the large contact forces. Note that in general, these forces may occur at both contact points, however one must check that vertical projections of these forces are positive throughout the collision. If any of these forces becomes negative, it means that the corresponding end of the dimer leaves the plate, and the contact forces acting on this end, vanish.

We assume that double collision begins with non-zero sliding velocity \( (U_b^l = U_{pb}^b \neq 0) \).

The equation of motion for the two contact velocities read

\[
\begin{align*}
md\mathbf{V}_l &= \sum_{c \in \{l,r\}} [d\mathbf{p}_c + I^{-1}(\mathbf{r}_c \times d\mathbf{p}_c) \times \mathbf{r}_l] \\
md\mathbf{V}_r &= \sum_{c \in \{l,r\}} [d\mathbf{p}_c + I^{-1}(\mathbf{r}_c \times d\mathbf{p}_c) \times \mathbf{r}_r]
\end{align*}
\]
Depending on the initial velocities and the dimer aspect ratio, there may be several possible scenarios of double collision. If $\hat{\mu} < 4R/5L$ and $j < 0$, the left end of the dimer does not acquire a finite impulse during the double collision, and the formulas for single collisions (5)-(16) can be used with $X = L/2, Z = -R$. Substituting $A = L/2R$ in (13) we obtain the condition $\hat{\mu}_s(A^2 + 7/5) > A$ for the transition from double slip-stick to double slip reversal, and using Eqs.(14)-(16) we recover formula (2) from the paper.

If $\hat{\mu} > 4R/5L$ and $j < 0$, during the initial slide the left ball also acquires a finite impulse. If

$$j \left[ U_0 - \frac{(Z - j\hat{\mu}X)}{2X} V_0(1 + \epsilon) \right] > 0$$

(35)

the negative slide continues until the right ball lifts off, and the horizontal CP velocity at the end of collision

$$\dot{U}_f = U_r^b - \frac{(Z - j\hat{\mu}X)}{2X} (1 + \epsilon) V_r^b.$$  

(36)

If condition (35) is violated, the sliding stops during double collision, and the frictional force reverses direction. If $XZ + j\hat{\mu}_s(Z^2 + K^2) < 0$, the contacts stick, and at the end of the collision

$$U_r^a = U_r^a = 0$$

(37)

$$V_r^a = \frac{K^2 + X^2 - Z^2}{K^2 + X^2} \left[ \frac{2X}{Z - j\hat{\mu}X} U_0 - (1 + \epsilon) V_0 \right]$$

(38)

$$V_r^a = -\epsilon V_r^b$$

(39)

If $XZ + j\hat{\mu}_s(Z^2 + K^2) > 0$, the slide reverses, and at the end of the collision

$$U_r^a = U_r^a = \frac{XZ + j\hat{\mu}(Z^2 + K^2)}{K^2 + X^2 + j\hat{\mu}XZ} \left[ -(1 + \epsilon) V_r^b + \frac{2X}{Z - j\hat{\mu}X} U_r^b \right]$$

(40)

$$V_r^a = \frac{(K^2 - X^2 - j\hat{\mu}XZ)}{K^2 + X^2 + j\hat{\mu}XZ} \left[ -(1 + \epsilon) V_r^b + \frac{2X}{Z - j\hat{\mu}X} U_r^b \right]$$

(41)

For the center of mass velocities

$$u^a = Z (V_r^a + \epsilon V_r^b) / 2X + U_r^a$$

(42)

$$v^a = (V_r^a - \epsilon V_r^b) / 2$$

(43)

$$\omega^a = (V_r^a + \epsilon V_r^b) / 2X$$

(44)

### V. EVENT-DRIVEN ALGORITHM

Using formulas presented in previous section, we can describe the motion of a dimer as a sequence of events which include collisions, periods of free flight, and rolling on the plate. In this Section we describe our event-driven algorithm. Here we denote the coordinates of the center of mass of the dimer at the $n$-th collision $\{x_n, z_n\}$, the tilt $\phi_n$, the CM velocities $\{v_n, \omega_n\}$, and the angular velocity $\omega_n$.

CM coordinates of the dimer during flight are denoted $\{x, z\}$, and coordinates of the centers of the left and right balls are $\{x_{l,r}, z_{l,r}\}$. The instantaneous angle of the dimer with vertical is $\phi$. The angular velocity of the dimer $\omega_n$ is conserved during flight.

We assume that the plate oscillates with amplitude $a$ and frequency $\Omega = 2\pi f$, $z_{pl} = a \sin(\Omega t)$.

Coordinates of CM during flight after $n$th collision:

$$x(t) = x_n + u_n(t - t_n); \quad z(t) = z_n + v_n(t - t_n) - g(t - t_n)^2/2; \quad \phi(t) = \phi_n + \omega_n(t - t_n).$$

(45)

The coordinates of the left and right ends of the dimer after $n$th collision:

$$x_{l,r}(t) = x(t) \pm \sin(\phi_n + \omega_n(t - t_n))L/2; \quad z_{l,r}(t) = z(t) \pm \cos(\phi_n + \omega_n(t - t_n))L/2$$

(46)

Here $t_n$ indicates the time of the $n$th collision. It is easy to see that $z_n = z_{pl}(t_n) + L \cos(\phi_n)/2$. The condition for the next collision is $z(t_{n+1}) = z_{pl}(t_{n+1})$ or $z(t_{n+1}) = z_{pl}(t_{n+1})$ whichever comes first. Thus, the smallest root $\tau_n$ of transcendental equations

$$L \cos(\phi_n)/2 + v_n \tau_n - g \tau_n^2/2 \pm \cos(\phi_n + \omega_n \tau_n) L/2 = a \sin(\Omega(t_n + \tau_n))$$

(47)
gives the time $t_{n+1}$ of the next collision: $t_{n+1} = t_n + \tau_n$. Corresponding coordinates are given by

$$x_{n+1} = x_n + u_n\tau_n; \quad z_{n+1} = z_n + v_n\tau_n - g\tau_n^2/2; \quad \phi_{n+1} = \phi_n + \omega_n\tau_n.$$  

(48)

The CM velocities after the next collision depend on which end of the dimer collides with the plate. For the left end colliding with the plate, $z_l(t_{n+1}) = z_{pl}(t_{n+1})$, we get

$$u_{n+1} = F_u(u_n, v_n - g\tau_n, \omega_n, \phi_n + \omega_n\tau_n)$$  

(49)

$$v_{n+1} = F_v(u_n, v_n - g\tau_n, \omega_n, \phi_n + \omega_n\tau_n) + a\Omega \cos(\Omega t_{n+1})$$  

(50)

$$\omega_{n+1} = F_{\omega}(u_n, v_n - g\tau_n, \omega_n, \phi_n + \omega_n\tau_n)$$  

(51)

where functions $F_u$, $F_v$ and $F_{\omega}$ are given by the above formulas (6)-(16). For the case of $z_r(t_n) = z_{pl}(t_n)$, in these expressions variable $X$ has to be taken in the form $X = -L \sin \phi/2$.

If $\Gamma < 1$, in certain cases one of the ends of the dimer exhibits an infinite sequence of ever-diminishing bounces off the plate which eventually leads to that end sticking to the plate in a finite time (an analog of an inelastic collapse). In order to circumvent this difficulty in numerical simulations, we impose a small threshold velocity $V_{th}$, so that if $V_c < V_{th}$, we set $V_c = 0$ and commence the rolling phase (Sec.III). The value of $V_{th}$ is not important and does not affect the dynamics of the dimer.

The rolling phase must end with a double collision which, as we have seen in Sec. IV, usually takes the form of double slip-stick. As a result, both balls of the dimer leave the plate and another sequence of single collisions begins.