I. CLASSIFICATION OF TOPOLOGICAL INSULATORS

The full classification (periodic table) of topological insulators and superconductors for all ten symmetry classes was developed in Refs. 2 and 3. This classification determines whether the Z or Z_2 topological insulator is possible in the d-dimensional system of a given symmetry class. Once the symmetry class allows the existence of a topological insulator, the emergence of the topological order in a particular system can be verified by inspecting the topology of (i) classifying space R_p; other π_d are given by

$$\pi_d(R_p) = \pi_0(R_{p+d})$$

The homotopy groups π_d have periodicity 8 (Bott periodicity).

There are two ways to detect topological insulators: by inspecting the topology of (i) classifying space R_p or of (ii) the sigma-model space S_p.

(i) Existence of topological insulator (TI) of class p in d dimensions is established by the homotopy group π_0 for the classifying space R_{p-d}:

$$\begin{cases} 
\text{TI of the type } \mathbb{Z} & \iff \pi_0(R_{p-d}) = \mathbb{Z} \\
\text{TI of the type } \mathbb{Z}_2 & \iff \pi_0(R_{p-d}) = \mathbb{Z}_2
\end{cases}$$

(ii) Alternatively, the existence of topological insulator of symmetry class p in d dimensions can be inferred from the homotopy groups of the sigma-model manifolds, as follows:

$$\begin{cases} 
\text{TI of the type } \mathbb{Z} & \iff \pi_d(S_p) = \mathbb{Z} \\
\text{TI of the type } \mathbb{Z}_2 & \iff \pi_{d-1}(S_p) = \mathbb{Z}_2
\end{cases}$$

This criterion is obtained if one requires existence of “non-localizable” boundary excitations. This may be guaranteed by either Wess-Zumino term in d-1 dimensions [which is equivalent to the Z topological term in d dimensions, i.e. \(\pi_d(S_p) = \mathbb{Z}\)] for a QHE-type topological insulator, or by the \(\mathbb{Z}_2\) topological term in d-1 dimensions [i.e. \(\pi_{d-1}(S_p) = \mathbb{Z}_2\)] for a QSH-type topological insulator.

The above criteria (i) and (ii) are equivalent, since

$$\pi_d(S_p) = \pi_d(R_{4-p}) = \pi_0(R_{4-p+d})$$

For the classification of topological insulators it is important to know homotopy groups π_d for all symmetry classes. In Table I we list π_0(R_p); other π_d are given by

$$\pi_d(R_p) = \pi_0(R_{p+d})$$

The homotopy groups π_d have periodicity 8 (Bott periodicity).

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$$\pi_d(S_p) = \pi_d(R_{4-p}) = \pi_0(R_{4-p+d})$$
TABLE I: Symmetry classes and “Periodic Table” of topological insulators\textsuperscript{2,3}. The first column enumerates the symmetry classes of disordered systems which are defined as the symmetry classes \( H_p \) of the Hamiltonians (second column). The third column lists the symmetry classes of the classifying spaces (spaces of reduced Hamiltonians)\textsuperscript{2}. The fourth column represents the symmetry classes of a compact sector of the sigma-model manifold. The fifth column displays the zeroth homotopy group \( \pi_0(R_p) \) of the classifying space. The last four columns show the possibility of existence of \( \mathbb{Z} \) and \( \mathbb{Z}_2 \) topological insulators in each symmetry class in dimensions \( d = 1, 2, 3, 4 \).

<table>
<thead>
<tr>
<th>( p )</th>
<th>Symmetry classes</th>
<th>( R_p )</th>
<th>( S_p )</th>
<th>( \pi_0(R_p) )</th>
<th>Topological insulators</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>AI</td>
<td>BDI</td>
<td>CII</td>
<td>( \mathbb{Z} )</td>
<td>0 0 0 ( Z )</td>
</tr>
<tr>
<td>1</td>
<td>BDI</td>
<td>BD</td>
<td>AII</td>
<td>( \mathbb{Z}_2 )</td>
<td>0 0 0 0</td>
</tr>
<tr>
<td>2</td>
<td>BD</td>
<td>DIII</td>
<td>DIII</td>
<td>( \mathbb{Z}_2 )</td>
<td>0 ( \mathbb{Z}_2 )</td>
</tr>
<tr>
<td>3</td>
<td>DIII</td>
<td>AII</td>
<td>BD</td>
<td>0</td>
<td>( \mathbb{Z}_2 ) 0 ( \mathbb{Z}_2 ) ( Z )</td>
</tr>
<tr>
<td>4</td>
<td>AII</td>
<td>CII</td>
<td>BDI</td>
<td>( \mathbb{Z} )</td>
<td>0 ( \mathbb{Z}_2 ) ( \mathbb{Z}_2 ) ( \mathbb{Z} )</td>
</tr>
<tr>
<td>5</td>
<td>CI</td>
<td>C</td>
<td>AI</td>
<td>0</td>
<td>( \mathbb{Z} ) 0 ( \mathbb{Z}_2 )</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>CI</td>
<td>C</td>
<td>0</td>
<td>( \mathbb{Z} ) 0 ( \mathbb{Z}_2 )</td>
</tr>
<tr>
<td>7</td>
<td>CI</td>
<td>AI</td>
<td>CI</td>
<td>0</td>
<td>( \mathbb{Z} )</td>
</tr>
</tbody>
</table>

and

\[
\pi_0(R_p) = \begin{cases} \mathbb{Z} & \text{for } p = 0, 4, \\ \mathbb{Z}_2 & \text{for } p = 1, 2. \end{cases}
\]

(6)

In this paper we focus on 2D systems of symplectic (AII) symmetry class. One sees that this is the only symmetry class out of ten classes that supports the existence of \( \mathbb{Z}_2 \) topological insulators both in 2D and 3D.

The effect of interaction on \( \mathbb{Z}_2 \) topological insulators and superconductors in classes DIII (in 2D) and CII (in 3D) will be considered elsewhere.

II. SURFACE STATES OF 3D TOPOLOGICAL INSULATORS

In this section we consider the surface of 3D topological insulators and derive the effective surface Hamiltonian (cf. an early work by Volkov and Pankratov, Ref. 6). The realistic microscopic Hamiltonian of such systems with strong spin-orbit interaction can be modelled by the 3D massive Dirac Hamiltonian, see, e.g., Refs. 3,5 and 6. We start with the general form of a 3D Dirac Hamiltonian (hereafter \( v = 1 \))

\[
H_{3D} = \begin{pmatrix} -M & \sigma p \\ \sigma p & M \end{pmatrix}.
\]

(7)

This Hamiltonian is a matrix in the \( 4 \times 4 \) space formed by the spin and pseudospin (sublattice) spaces. The interface between the semiconductor and the vacuum is described by sending the mass \( M \) to infinity.

Let us consider a flat interface at \( x = 0 \). The edge state with zero energy decays into the bulk (\( x < 0 \)):

\[
\Psi = e^{-Mx} \begin{pmatrix} \psi \\ \chi \end{pmatrix}.
\]

(8)

Acting by the Hamiltonian (7) on this wavefunction, we obtain the relation between the two components of the spinor, \( \chi = i\sigma n\psi \). For an arbitrary surface characterized by the normal vector \( n \), the general boundary condition reads

\[
\chi = i\sigma n\psi.
\]

(9)

From Eqs. (7) and (9) we obtain the effective 2D surface Hamiltonian in the form of the Rashba Hamiltonian in a curved space

\[
H_{surf} = \frac{i}{2} [\sigma p, \sigma n] = \frac{\nabla n}{2} + \frac{1}{2} (n[p \times \sigma] + [p \times \sigma]n).
\]

(10)

This 2D Hamiltonian describes a single species of 2D massless Dirac particles (cf. Ref. 6) and is thus analogous to the Hamiltonian of graphene with just a single valley.

III. ABSENCE OF LOCALIZATION OF 2D SURFACE STATES

Consider the 2D system formed at the surface of a 3D topological insulator and described by the Hamiltonian (10). Let us first prove the absence of localization in the non-interacting case. Assume that all the states in 2D are localized with some localization length \( \xi \). Consider a hollow cylinder with all dimensions much larger than \( \xi \) pierced by the Aharonov-Bohm magnetic flux \( \Phi = hc/2e \).
FIG. 1: Left: schematic illustration of the hollow cylinder sample used for proving the absence of localization on the surface of a 3D topological insulator. Right: the energy spectra of a clean 2D system on the surface of the cylinder with zero flux and with half of the magnetic flux quantum penetrating the cylinder.

(half of the flux quantum), Fig. 1. This value of \( \Phi \) does not break the time-reversal symmetry leaving the system in the symplectic class. In the absence of disorder we can characterize the surface states by the momentum \( k \) along the cylinder axis and by the integer angular momentum \( n \). The energy of such a state is given by (see Fig. 1)

\[
E = \sqrt{k^2 + (n/R)^2}.
\]

The channels with positive and negative \( n \) are degenerate, while \( n = 0 \) channel is not. Thus the cylinder sustains an odd number of conducting channels both on the inner and outer surface at any value of chemical potential.

Let us now include disorder and show the absence of localization in quasi-one-dimensional (q1D) symplectic system with odd number of channels. The scattering matrix of such a q1D wire has the form

\[
S = \begin{pmatrix} r & r' \\ t & t' \end{pmatrix}
\]

with transmission and reflection amplitudes as its entries. The blocks \( r \) and \( r' \) are square matrices of the size determined by the number of channels. Time-reversal symmetry of the symplectic type imposes the following restrictions on the amplitudes entering the matrix \( S \):

\[
r = -r^T, \quad r' = -r'^T, \quad t = t'^T.
\]

Calculating the determinant of the both sides of the first identity and taking into account the odd size of the matrix \( r \), we obtain \( \det r = 0 \). This implies a zero eigenvalue of \( r \) and hence the existence of a channel with perfect transmission. We conclude: in a q1D wire of symplectic symmetry with an odd number of channels one channel always remains delocalized.

Applying the q1D result to the cylinder constructed above we immediately come to the controversy: in spite of assumed 2D localization on the surface, the infinitely long cylinder possesses two (inner and outer) conducting channels. This proves the absence of localization in 2D.

The proof can be generalized to include the Coulomb interaction. We assume the temperature to be much smaller than the inverse time of electron propagation through the system. At such low temperatures the inelastic scattering of electrons is negligible and we can describe the transport by the single-particle scattering matrix (12) calculated at the Fermi energy and accounting for virtual processes. The latter renormalize the parameters of the \( S \) matrix, the renormalization being cut off by the system size. Assuming no interaction-induced breaking of the time-reversal symmetry (see Sec. IV), the symmetry properties of this \( S \) matrix are unchanged and hence the above proof applies.

The persistence of topological protection of 2D surface states in the presence of interaction is encoded in the structure of the replicated Matsubara sigma-model. Similarly to the ordinary QHE, this theory possesses the same nontrivial topology as in the non-interacting case.

IV. TOPOLOGICAL PROTECTION IN THE PRESENCE OF INTERACTION

In this section we discuss the stability of topological insulators with respect to electron-electron interaction. We are going to demonstrate that not too strong interactions do not affect the existence of the topological-insulator phase.

The topological insulators are characterized by a certain topological invariant related to the symmetry of the problem (see, e.g. Ref. 4 for the general case when both interaction and disorder are included). Therefore, in order to destroy the topological nature of the insulator, the interaction should change the symmetry properties of the ground state. This is, in principle, possible for sufficiently strong interaction. Indeed, it is well known that strong interaction may lead to instabilities and sponta-
neous symmetry breaking, thus giving rise to new phases. An example of such interaction-induced new phase is the ferromagnetic phase driven by the interaction-induced Stoner instability. Another well-known example, related to topological insulators, is the fractional quantum Hall effect state.

In the present case of time-reversal invariant $\mathbb{Z}_2$ topological insulators, we thus have to examine possible interaction-induced instabilities in both clean and disordered systems. These instabilities would reveal themselves in the properties of the boundary states at the surface of topological insulators.

We start with the 2D surface of a 3D topological insulator. In this paper, we are interested in the effect of long-ranged Coulomb interaction on the scales exceeding the mean-free path $\ell$, where we employ our diffusive RG treatment of the interaction effects. This treatment is justified if there are no interaction-driven instabilities at the ballistic scales $L \ll \ell$.

For example, it might happen that already in the ballistic regime an instability of a Stoner type would occur which would spontaneously break the time reversal symmetry. In this case, a very strong interaction could drive the surface electrons ferromagnetic, so that the surface states would become fully gapped. This would place the disordered problem into the unitary symmetry class, which does not support 3D topological insulators. Then our diffusive RG designed for the symplectic symmetry class would not be applicable. Therefore, let us first examine the ballistic regime.

First of all, the spin-orbit interaction, which yields the peculiar band structure of the $\mathbb{Z}_2$ topological insulator, is characterized by the length scale $l_{SO}$. For very short scales $L \ll l_{SO}$ the spin of electrons is effectively conserved and the triplet channel is not yet suppressed by the spin-orbit coupling. A Stoner instability due to very strong interaction in the triplet channel may then compete with the spin-orbit interaction and establish the gapped phase on the scale $L \ll l_{SO}$. This, however, would just imply that the system under study is not a topological insulator: the interaction is so strong that it fully changes the band structure of the system. We assume that this is not the case so that on the scale of $l_{SO}$ we have a topological insulator.

For $L \gg l_{SO}$, the surface states of a topological insulator are described by Dirac fermions, see Sec. II (with $l_{SO}$ serving as an ultraviolet cutoff). The problem of interacting Dirac fermions arising for the surface of a 3D topological insulator is analogous to that for a single flavor of Dirac particles in graphene. The latter has been analyzed in detail in a number of works (see Ref. 9 for review). As far as the interaction effects are concerned, the main difference between clean (or ballistic) and disordered (diffusive) graphene is the screening of the Coulomb interaction. Indeed, the density of states participating in the screening is drastically affected by disorder. It linearly depends on energy in the clean case (hence screening is suppressed) whereas the scalar potential induces a constant density of states near the Dirac point (hence diffusive screening).

In the context of clean graphene, it has been found that sufficiently strong Coulomb interaction between Dirac fermions may indeed open a gap in the spectrum, leading to the spontaneous breaking of time-reversal symmetry. This happens for the large values of interaction parameter $r_s = \sqrt{2}\epsilon^2/\hbar v_F$ (where $\epsilon$ the dielectric constant of the environment). However, for $r_s \ll 1$ the interaction constant is marginally irrelevant\cite{9,10}, since the Fermi velocity increases upon the ballistic renormalization. Therefore, for not too strong Coulomb interaction there is nothing special in the clean case: no instabilities are expected. This conclusion remains intact also in the presence of scalar potential due to impurities in the ballistic regime $L \ll \ell$.

Once we have no interaction-driven instabilities for $L \ll \ell$ (which is the case for $r_s \ll 1$), we can safely turn to the diffusive scales $L \gg \ell$ and employ the diffusive RG approach\cite{11} which describes the interplay between Coulomb interaction and disorder. This diffusive RG may lead to the instability of the Stoner type for the case of several species of particles ($N > 1$), since disorder enhances the interaction in the multiplet channel. Importantly, there is no renormalization of long-range Coulomb interaction for a single flavor ($N = 1$). This is because for the long-range (unscreened by external gates) Coulomb interaction the internal screening at small momenta renders the effective interaction constant universal, see Sec. V. This property is based on the singularity of the Coulomb interaction in the long-wavelength limit. Thus, in the case of a 2D surface of a 3D topological insulator the RG equations yield no instability.

Let us now turn to the case of a 1D surface (edge) of 2D QSH topological insulator. In the absence of impurities the interacting electrons on the edge form a clean Luttinger liquid. The conductance of the edge is therefore independent of the interaction. This means that even the arbitrary strong interaction can not destroy the 2D QSH topological insulator phase when the 1D edge is clean.

In the presence of impurities, the situation is more intricate. The single-particle backscattering of impurities is prohibited by the time-reversal symmetry. Indeed, the edge states with opposite momenta correspond to the orthogonal spin states due to the Kramers degeneracy. Therefore, the disordered 1D systems seems to behave as a clean Luttinger liquid with delocalized edge modes.

However, as has been found in Refs. 12 and 13 (see also Refs. 14 and 15) the two-particle backscattering (2PB) processes and, more generally, backscattering processes involving an even number of particles do not violate the time-reversal symmetry. The 2PB process are analogous to the local Umklapp scattering. In the case of randomly distributed impurities, these processes can be thought of as a very weak fluctuating Umklapp.

For a single impurity, the coupling constant describing the 2PB (as well as those describing the higher-order backscattering) is irrelevant for weak interaction. How-
ever, for $K < 1/4$ (here $K$ is the Luttinger parameter describing the strength of the short-range interaction; $K = 1$ for non-interacting particles) this coupling increases under RG, leading to the pinning of the bosonic excitations at the impurity cite.

In the presence of quenched disorder, the bosonized action including the 2PB is identical (upon rescaling the bosonic fields by a factor of four) to the action of disordered spinless electrons experiencing the ordinary backscattering. The ballistic RG equation describing the renormalization of the dimensionless strength of the 2PB disorder $D_2$ is analogous to that derived in Ref. 16 with the replacement $K \rightarrow 4K$:

$$\frac{dD_2}{d\ln L} = (3 - 8K)D_2.$$  

(14)

This equation implies that for not too strong interaction the coupling $D_2$ is irrelevant, whereas for sufficiently strong interaction, $K < 3/8$, $D_2$ processes become relevant. The interaction constant $K$ is itself also the subject of the renormalization in the presence of quenched disorder.$^16$

For the case of long-range Coulomb interaction, since the interaction matrix element logarithmically depends on the transferred momentum $q$, the bare Luttinger parameter is $q$-dependent:

$$K_0(q) = \left(1 + 2\alpha \ln \frac{q_0}{q}\right)^{-1/2},$$

(15)

where $\alpha = e^2/2\pi\hbar v_F = r_s/2\sqrt{2}\pi$ and $q_0$ is the ultraviolet scale. According to Eq. (15), the interaction parameter eventually becomes smaller than $3/8$ at a certain large scale, even without the renormalization by disorder. This suggests that for unscreened Coulomb interaction, the two-particle backscattering becomes strong for arbitrary weak interaction parameter $\alpha$. Let us estimate the corresponding length scale.

For simplicity, we neglect the renormalization of the interaction parameter by disorder and plug in the bare interaction parameter from Eq. (15) into Eq. (14):

$$\frac{dD_2}{d\ln L} = \left[3 - \frac{8}{(1 + 2\alpha \ln L)^{1/2}}\right]D_2,$$  

(16)

where $L$ is measured in units of $q_0^{-1}$ (which in experimental situation$^{17}$ is given by the width of the 1D channel $\sim 10$ nm). The solution of this equation reads:

$$D_2 = D_0^2 L^3 \exp\left\{\frac{8}{\alpha}(1 + 2\alpha \ln L)^{1/2} - 1\right\}.$$  

(17)

For small $\alpha \ll 1$, the disorder coupling decreases upon renormalization and reaches its minimal value at $L_{\text{min}} = \exp(55/18\alpha)$. At larger scales, the 2PB disorder starts increasing, reaches its initial value $D_0^2$ at

$$L_0 = \exp(80/9\alpha),$$  

(18)

and becomes of order of one at

$$L_1 \sim (D_0^2)^{-1/3} \exp\left[\frac{40}{9\alpha} \left(1 + \sqrt{1 - \frac{6\alpha}{25} \ln D_0^2}\right)\right].$$  

(19)

For $r_s \lesssim 1$ the scale $L_0$ is unrealistically large: for $r_s = 1$ we get $\alpha \approx 0.11$ and $L_0 \sim 10^{14}$. For $v_F \sim 5 \times 10^5$ m/s as in HgTe QSH edge$^{17}$, even if we assume $\epsilon = 1$ we get $r_s \sim 5$, yielding $L_0 \approx 4 \times 10^8$, which translates into the system size of the order of few meters. Taking larger $\epsilon$ as expected for experimental conditions of Ref. 17 suppresses $r_s$ and further exponentially enhances $L_0$. Therefore, for a not too strong Coulomb interaction the topological character of the finite-size 2D QSH insulator is not destroyed. What happens for sample sizes larger than $L_0$ (in particular, how the renormalization of $D_2$ affects the properties of the topological insulators) requires further study. This question is, however, of purely academic interest if interaction is not too strong.

Finally, it is worth considering the general model of a long-range interaction potential $V(r - r') = V_0 |r - r'|^{-\gamma}$ with $0 < \gamma < 1$ (the Coulomb interaction corresponds to $\gamma = 0$). In this case, the 1D Fourier component of the interaction is finite at $q \rightarrow 0$, whereas the 2D interaction remains singular. This implies that the 2D system with such interaction behaves analogously to the Coulomb case, whereas the 1D system is characterized by a momentum independent parameter $K$. For not too large $V_0$ one has $K > 3/8$ and the 2PB is irrelevant. Thus our conclusions for 2D topological insulators are fully applicable.

V. RENORMALIZATION-GROUP FOR INTERACTING 2D SYSTEMS OF SYMPLECTIC SYMMETRY CLASS

In this section we analyze the RG equations for interacting disordered systems belonging to the symplectic symmetry class for the case of long-range Coulomb interaction as well as for short-ranged interaction. The sigma-model RG approach to interacting disordered systems is pioneered by Finkelstein (see Ref. 11 for review). The one-loop RG equations are fully controllable in the range of large conductance $g \gg 1$. All expansions for beta function and related quantities go in small parameter $1/g$, while the interaction is accounted for to all orders. Since we are interested in the case of single flavor, $N = 1$, we do not include the multiplet term into consideration from the very beginning.

We first consider the case of long-range Coulomb interaction. The RG equations in the symplectic class have been derived in Ref. 19 for systems with strong spin-orbit coupling. In addition to the universal term generated by Coulomb interaction in the singlet channel, the set of equations contains the singlet interaction amplitude in
the particle-particle (Cooper) channel $\gamma_c$:

\[
\frac{dg}{d \ln L} = -\frac{1}{2} + \gamma_c, \quad (20)
\]

\[
\frac{d\gamma_c}{d \ln L} = \frac{1 + \gamma_c}{2g}, \quad (21)
\]

\[
\frac{d \ln Z}{d \ln L} = -\frac{1 - 2\gamma_c}{2g}. \quad (22)
\]

Here $L$ is the running RG scale, e.g., the system size. The term $-1/2$ in Eq. (20) is the difference of contributions of weak antilocalization and singlet term of interaction in particle-hole channel. Equation (22) describes the renormalization of temperature scale. Indeed, for small starting values of $\gamma_s$ and $\gamma_c$ we see from Eq. (23) that the conductivity increases due to the weak antilocalization. According to Eq. (25), the Cooper amplitude is attracted by $\gamma_c = (\gamma_s/2g)^{1/2}$. This, being inserted into Eq. (24), yields $\gamma_s \to 2/g \ll 1$ and hence $\gamma_c \to 1/g \ll 1$. Thus both interaction amplitudes decrease and cannot affect the increase of $g$. Therefore, for sufficiently weak short-range interaction the supermetallic fixed point remains stable and the critical state does not develop.

For strong short-range interaction the situation may be more intricate. It is known\textsuperscript{18,20} that strong particle-hole singlet amplitude for the case of arbitrary ($\gamma_s < 1$) short-range interaction decreases according to

\[
\frac{d\gamma_s}{d \ln L} = -\frac{\gamma_s}{2g}(1 - \gamma_s) \quad (27)
\]

(here and below we omit the irrelevant contribution of the interaction in the Cooper channel). For initially strong $\gamma_s$, the singlet particle-hole term\textsuperscript{18,20} overcomes the weak antilocalization in $\beta(g)$:

\[
\frac{dg}{d \ln L} = \frac{1}{2} - \frac{1 - \gamma_s}{\gamma_s} \ln(1 - \gamma_s), \quad (28)
\]

so that the conductivity starts flowing to smaller $g$ and may reach $g \sim 1$. The further analysis requires the understanding of the RG equation in the range of strong coupling. This is, however, beyond the scope of this work.

\textsuperscript{5} L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. 98, 106803 (2007); L. Fu and C. L. Kane, Phys. Rev. B 76, 045302 (2007).