I. DEVICE FABRICATION

The suspended samples were fabricated on single crystalline (001) Si wafers with double-sided 30 nm thick low-stress silicon nitride (SiN) films grown by low-pressure chemical vapour deposition (LPCVD) and purchased from the Microfabrication Laboratory at UC Berkeley. A 500 × 500 μm square was first patterned and etched onto the bottom-side nitride, using photolithography and reactive ion etching in CHF₃ gas. The remaining SiN then served as an etch mask for a crystallographic potassium hydroxide (KOH) wet etch through the whole wafer thickness, producing a 30 nm thick suspended SiN membrane of size 50 × 50 μm on the front side (bulk micromachining). The metallic wires were then deposited on the membrane, using electron beam lithography and two-angle shadow mask technique, where the two metals aluminium (Al) and copper (Cu) were e-beam evaporated from different angles in an UHV system with a base pressure of 10⁻¹¹ mbar. Al was evaporated first (0.2 nm/s) at 60° angle with respect to the normal of the substrate, after which it was thermally oxidized in 10 mbar for 4 min. Then Cu was evaporated with rate 0.15 nm/s from 0° angle. This produced (after lift-off) the narrow nanowire, the smaller Al/AlOₓ/Cu thermometer junctions (typical RT ~ 30 – 50 kΩ) in the middle of the wire, and the larger Al/AlOₓ/Cu cooler junctions (typical RT ~ 1.5 – 3 kΩ) at the ends of the wire (Fig. 1(c), main text). After metallization, the suspended structure was released by reactive ion etching the SiN in a CHF₃ plasma (0.1 mbar for 100 s), where the metal wires themselves served as the etching mask. Note that this process also etches down the SiN layer on the bulk substrate area around the wider leads.

The bulk samples were fabricated on the same 30 nm LPCVD-nitridized Si wafers during the same evaporation runs, but without any etching processes. Samples used in the heating experiment containing direct superconductor–normal (SN) junctions had a more complex fabrication procedure than the coolers, mainly because three different materials were used: Cu as the normal metal, Al as the superconductor for NIS thermometers and Nb as the superconductor for the heating probes connecting directly to Cu. The thermometer junctions were still the same Al–AlOₓ–Cu SiN junctions as in the coolers, and thus the first evaporation and oxidation steps of Al were the same as for the cooler samples, (60° angle, thermal oxidation in 10 mbar for 4 min). Then the sample stage was rotated horizontally by 90° and Cu deposited from tilt–angle of 60°. After that, the sample was again rotated horizontally by 45° and deposition of 30 nm of Nb with rate of 0.5 nm/s followed from the tilt–angle of 60°. The structural release process was the same as for the suspended coolers. A Schematic view of the resulting heating sample is shown in Fig. 1. Nb was used for the heating junctions because of its high superconducting gap, which prevents any heat from leaking into the superconductor due to multiple Andreev reflections [1]. This means that the dissipated power within the normal metal nanowire can be accurately determined by simply measuring the IV characteristics of the SNS structure, and calculating the dissipated power as P_{diss} = IV.

II. OPERATION OF THE COOLER

The basic principle of cooling of a NIS tunnel junction is based on the existence of the superconducting energy gap Δ for single particle electronic excitations (Fig. 1(a), main text). An electron from the normal metal cannot enter the superconductor, unless it has at least energy Δ. A voltage bias V can supply this energy so that at T = 0 current can flow if eV > Δ. However, at finite temperatures electrons follow the Fermi-Dirac distribution, so that even at biases eV < Δ there are some energetic (hot) electrons that can tunnel. As only hot electrons escape in that case, the temperature of the remaining electrons in the normal metal is lowered, and the magnitude of bias dependent heat flow (power) due to single particle tunneling from the normal metal to the super-
The main text discussed the differences between bulk and suspended samples in terms of measurements, where the cooler bias was kept at optimum value, and bath temperature was varied (Fig. 2, main text). A further confirmation of the difference between bulk and suspended samples is shown in Fig. 2, where measured cooling curves ($T$ vs. $V$) for the bulk (dashes, black line) and the suspended (solid, red line) sample of similar values of $R_T$ are plotted for three different bath temperatures, and compared with the thermal model (Eq. 1 of main text) limited either by 3D or 1D electron–phonon (e-p) interaction in the Cu film (open circles and diamonds, respectively) using typical values $n = 5$ ($n = 3$ for 1D) and $\Sigma = 2.1 \cdot 10^9$ W K$^{-5}$ m$^{-3}$. In addition, for the 1D model we used $c_2 = 4900$ m/s for the longitudinal speed of sound in Cu. The only fitting parameter used was $\beta$, the fraction of dissipated heat flowing back to the normal metal, with a value $\beta = 0.03$ kept constant for the three bath temperatures and for both models. As we see, the 3D e-p model reproduces the bulk data very well for all bath temperatures, but cannot explain the suspended sample data at all. We stress that since the electron gas volumes of the two samples are equal, there are no fitting parameters left, as we expect $\beta$ to be approximately the same for the bulk and suspended samples (both samples have the coolers on the bulk substrate). For the low temperature range, the suspended nanowire phonons are expected to be one-dimensional, thus we should also compare the suspended data with the one-dimensional e-p model [4] (diamonds, Fig. 2). It is clear that it cannot model the suspended data either (and adjusting $\beta$ does not improve the fit). The $T(V)$ cooling curves thus confirm the conclusion that e-p interaction is not the limiting dissipation mechanism in the suspended nanowires, unlike in the bulk samples.

IV. DETAILS ON SINIS THERMOMETER

In addition to using NIS junction as coolers, they were also used as sensitive thermometers because of their highly non-linear and temperature dependent current–voltage (I–V) characteristics at sub-Kelvin temperatures [2, 5]. In practice, the measurement is usually performed by connecting two junctions in series (SINIS), running a constant current through the junctions and measuring the temperature dependent voltage response. When constant current biased, the SINIS-thermometer voltage $V_{\text{therm}}$ is a sensitive function of temperature only, and this dependence can be calculated from the BCS–theory once the tunneling resistance of the junctions $R_T$ and the superconducting gap $\Delta$ are known. Since those parameters were always determined in a separate measurement of the I–V characteristics of the junctions, no free parameters are left, and the measured SINIS voltage can be unambiguously converted to temperature. We always checked this to be true by a calibration measurement, where the SINIS temperature was compared with the temperature given by a calibrated RuO thermometer while the refrigerator temperature was varied. The
thermometer junctions were current biased with a bias resistor $R = 10 \, \Omega$ to ensure proper current bias even in the subgap, where junction resistance was typically $\sim 10 \, \Omega$ at low temperatures. In the experiment, two different constant bias current values ($I \sim 10 \, \text{pA}$) and ($I \sim 100 \, \text{pA}$) were used, the low one for low-temperature regime and higher one for high-temperature regime. Two values were used because the SINIS temperature-to-voltage responsivity $dV/dT$ is a strong function of both $I$ and $T$ in such a way that the low bias (high bias) value gives a better responsivity at $T < 0.4 \, \text{K} (T > 0.4 \, \text{K})$. In addition, the lower bias value was always chosen higher than the measured two-electron/lifetime-broadened excess sub-gap current, to guarantee good response and no dependence on the details of the excess current mechanisms. $V\text{therm}$ was measured with a high input impedance differential voltage preamplifier (Ithaco 1201). In addition, while measuring the I–V characteristics, current was measured with a current preamplifier (Ithaco 1211).

It is clearly seen that by choosing the bias current appropriately (two examples shown as horizontal lines in Fig. 3), the responsivity $dV/dT$ of the thermometer can be adjusted to be best suited for the required temperature range. Fig. 4 shows an example of the bath–temperature-to-voltage response (the bath temperature was measured with a calibrated RuO thermometer) of the SINIS thermometer in Fig. 3 with bias current values 10 pA and 100 pA. With the low bias–current ($\sim 10 \, \text{pA}$) there is a gain in the responsivity at low temperatures, while with the higher bias–current ($\sim 100 \, \text{pA}$) the responsivity is better at higher temperatures $T > 0.4 \, \text{K}$. Best results with thermometry are obtained by repeating experiments with few different bias points for different temperature ranges. Note from Fig. 3 that at current levels $I \sim 1 \, \text{pA}$ temperature sensitivity is lost.

A. IV characteristics and subgap conductance of NIS junctions

The I–V characteristics can be understood by a simple BCS-based tunneling Hamiltonian theory [2, 6], which predicts for a pair of identical normal-metal-superconductor tunnel junctions (SINIS)

$$I(V) = \frac{1}{2\pi R_T} \int_{-\infty}^{\infty} g_S(E) [f_N(E-eV/2) - f_N(E+eV/2)] dE,$$

(2)

where $R_T$ is the tunneling resistance of a single junction, $f_N(E)$ is the Fermi-function of the normal metal and $g_S(E)$ is the quasiparticle density of states (DOS) in the superconductor. Note that equation (2) contains only the temperature of the normal metal and not that of the superconductor, and is valid as long as the quasiparticle distribution in the superconductor is also in quasiequilibrium (but not necessarily the same temperature). $g_S(E)$ can be determined from the BCS theory, which predicts in the weak coupling limit that $g_S(E) = |E|/\sqrt{E^2 - \Delta^2}$, where $|E| > \Delta$ and $g_S(E) = 0$ when $|E| < \Delta$ [6]. However, in real materials there are processes that create quasiparticle states also within the gap $|E| < \Delta$. The easiest and most straightforward way to model these is with the so called Dynes-parameter, which was initially realized to model life–time broadening due to inelastic scattering (electron-electron, electron-phonon) [7]. This leads into a DOS of the form

$$g_S(E) = \text{Re} \left\{ \frac{E + i\Gamma}{\sqrt{(E + i\Gamma)^2 - \Delta^2}} \right\},$$

(3)

where parameter $\Gamma$ describes the finite life–time ($\Gamma = \hbar/\tau$) of quasiparticle states in the superconductor.

Figure 5 shows a typical measured suspended SINIS cooler I–V curve (black, solid line) in a logarithmic scale to highlight the sub-gap regime. As the cooler junctions are larger and have thus lower $R_T$, the current is higher than in the thermometer junctions (Fig. 3). This means that the sub-gap current is more easily measurable. The dashed, red line in Fig. 5 is a theoretical curve based on Eq. 2 that takes into account the broadened DOS by the Dynes model (dashed, red line), providing a good fit to the data with $\Gamma/\Delta = 2 \cdot 10^{-4}$ as the only fitting parameter. This value is consistent with what has been observed before for thin aluminum films [7–9]. On the contrary, without the broadening (dashed, blue line) the sub–gap I–Vs can not be fitted. Cooling of the junctions (apparent as dip around $\sim 0.4 \, \text{mV}$) is taken into account in the theoretical curves by using a voltage-dependent temperature that was obtained from the separate thermometer junctions.

In addition to the finite life–time broadening, the microscopic nature of the finite sub–gap current can also...
FIG. 3: A measured sub-gap current–voltage characteristics of a typical SINIS thermometer \((2R_T = 63 \text{ k\Omega})\) at different bath temperatures in log-linear scale. Solid horizontal lines from top to down correspond to typical bias currents 100 pA and 10 pA, respectively.

FIG. 4: Measured voltage-bath temperature response of the SINIS thermometer in Fig. 3 with two different values of bias current, 10 pA (black line) and 100 pA (red line) (same values as the lines in Fig. 3). Open circles were calculated using the single-particle tunneling Hamiltonian, Eq. 2. Bath temperature was measured with a calibrated RuO thermometer.

FIG. 5: Sub-gap current voltage characteristics of SINIS Cooler junctions at \(T_{\text{bath}} = 60 \text{ mK}\). Experimental data is presented by a solid black line. Dashed lines show numerical calculations with broadened DOS \(\Gamma/\Delta = 2 \times 10^{-4}\) (red) and the case where life-time broadening is neglected (green). Inset shows the numerically calculated differential conductance \(dI/dV\) of the measured data (black) and theory with \(\Gamma/\Delta = 2 \times 10^{-4}\) (red). \(R_T = 3.3\text{k\Omega}\).

B. Temperature measurement in the low-temperature regime

As was shown above, Eq. 2 gives an accurate description of the measured \(I(V)\) curves, even at sub-gap voltages. This means that if we measure the \(I(V)\) curve of the SINIS thermometer, we can determine \(R_T, \Delta\) and \(\Gamma\) unambiguously, and use Eq. (1) to convert the measured SINIS voltage to temperature without fitting parameters. We also point out that by biasing clearly above the sub-gap knee (Fig. 3) \(\Gamma\) does not influence the results at all at the temperature range of interest, so that in practice only \(R_T\) and \(\Delta\) fix the temperature calibration. However, if one compares the measured responsivity curves \(V_{\text{SINIS}}\) vs. \(T_{\text{bath}}\) with the theory from Eq. 2 (Fig. 4), some deviation is clearly seen at the low temperature regime \(T < 100\text{mK}\), looking like a beginning saturation of \(V_{\text{SINIS}}\). This saturation is not an intrinsic limit for the junctions, since we have measured higher values of \(V_{\text{SINIS}}\) (lower temperatures) with the coolers operating. We conclude that the observed saturation is most likely caused by extrinsic heating power radiated down the leads. This is plausible since a nanoscale sample can be overheated very easily due to the weakness of dissipation (electron-phonon, or phonon scattering). From the heating experiment (Fig. 3(a) in the main text) we can easily see that for our devices an excess power levels of \(\sim 6 \text{ fW}\) is enough to heat up the sample to 100 mK from \(T_{\text{bath}} = 60 \text{ mK}\). This kind of power levels are easily caused by Johnson noise power radiated down from the 4 K stage filters within the bandwidth of our low-
temperature (below 4K) filters. However, the most important conclusion is that this observed saturation does not limit our temperature measurement for the coolers, as the theoretical self-calibration using Eq. 2 is still valid at $T < 100$ mK, regardless of the excess noise heating.

C. Lack of temperature gradients in the nanowire

FIG. 6: The cooler $I$–$V$ curve of the sample (black line), whose $T(V)$ cooling curves were shown in Fig. 3, main text. $T_{\text{bath}} = 60$ mK. Theoretical curves (Eq. 1) with $T$ as a parameter are also shown, with $T = 47.3$ mK giving the best fit at maximum cooling point around $V \sim 0.41$ V.

In the cooling experiments, we measured the temperature in the middle of the wire, whereas the coolers are actually located at the ends of the wire, a distance of $\sim 10$–15 microns away (Fig. 1(c), main text). One might wonder whether any temperature gradients will develop within the nanowire, and whether the measured temperature is therefore unequal to the temperature at the cooler junctions. We investigated this question by comparing the measured $I$–$V$ characteristics of the cooler junctions with theoretical $I$-V curves with temperature as a parameter, Fig. 6. It is clear that the cooler $I$–$V$ is consistent with lowest temperature $T = 47$ mK for this sample, which is the same temperature that was measured at optimal cooling for this sample at the SINIS thermometer in the middle of the nanowire (Fig. 4(b) main text). We conclude that no thermal gradients develop in the nanowire. This is consistent with the conclusion that phonon transmission at the nanowire–bulk boundary is the limiting dissipation mechanism.

D. Lack of influence of charging effects

Coulomb blockade (charging effects) due to the small capacitance of sub-micron scale tunnel junctions can have a measurable effect on thermometry, especially in the limit $E_C > k_B T$ [13], where $E_C = e^2/2C_\Sigma$ is the charging energy and $C_\sigma = 2C + C_0$ is the total capacitance of the junctions and island. $E_C$ can be experimentally determined by measuring the tunneling conductance spectrum around zero bias in the weak Coulomb blockade limit $E_C < k_B T$, where a dip $\Delta G$ develops, depending on $E_C$ as [2]

$$\Delta G = \frac{E_C}{3k_B T} \cdot (4)$$

where $G_T$ is the tunneling conductance around $V = 0$ without the dip. The measured charging energy at 4.2 K for a typical cooler sample is shown in Fig. 7. From the measurement we obtain $E_C/k_B = 20$ mK, which is small enough not to affect the SINIS thermometry, i.e. analysis can be carried out with the simplest BCS–theory calculation, Eq. (1), without charging effects. This conclusion has been confirmed by a theoretical calculation of the SINIS response including charging effects [13].

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