Supporting Information for

**Sheathless Electrokinetic Particle Separation in a Bifurcating Microchannel**

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Determination of Eq. 7 in the main text

The same approach as that used by Kang et al.\(^1\) was employed here. Specifically, we build a 3D model in COMSOL to directly calculate the wall-induced electrical lift force, \(F_w\), on a dielectric sphere with radius, \(a\), placed near one sidewall of a rectangular microchannel. The width (65 \(\mu\)m) and depth (40 \(\mu\)m) of the microchannel in the model are identical to those of the channel used in the experiment. The length of the channel is set to be 20\(a\) to reduce the computational time which is, however, still long enough to eliminate the channel end effects. The sphere is placed to the middle of the channel length with a fixed distance of 0.25 \(\mu\)m above the bottom channel wall according to Young and Li’s analysis.\(^2\) The separation gap between the sphere and one sidewall, i.e., \((h - a)\) in inset I of FIG. 2 in the main text, is varied from 0.05\(a\) to \((w/2 - a)\). The 3D electric field distribution, \(E\), around the sphere is first computed from solving Laplace’s equation. Then, the magnitude of the electrical lift force, \(F_w\), which is normal to the channel wall and points to the fluid, is obtained from the integration of the Maxwell stress tensor over the entire surface of the sphere,\(^3\)

\[
F_w = \int_S \varepsilon (EE - \frac{1}{2} E^2 I) \cdot n ds
\]  

(S-1)

where \(I\) is the unit tensor and \(n\) is the unit normal vector of the sphere. Next, the calculated \(F_w\) is non-dimensionalized by \(\varepsilon a^2\) and plotted against the non-dimensional gap between the sphere and sidewall, i.e., \(\gamma = (h - a)/a\) defined in Eq. 7 of the main text; see FIG. S-1. Following that the method of the least-square curve fitting (MATLAB) is used to fit the numerical data in FIG. S-1 to an exponential curve at the third order of \(\gamma\).

\[
F_w = C_0 \exp(C_1\gamma + C_2\gamma^2 + C_3\gamma^3)\varepsilon a^2 E^2 n
\]  

(S-2)
where the fitting constants $C_0$ to $C_3$ for 5 µm and 15 µm-diameter spheres are listed in Table 1 of the main text. We find that such an exponential function of a cubic polynomial can provide a much better fit to the numerical data in FIG. S-1 than that of a linear polynomial, especially accurate for near-wall particles. Finally, balancing $F_w$ in Eq. S-2 with Stokes’ drag yields the formula of $U_w$ in Eq. 7 of the main text.

**FIG. S-1.** Curve fitting (solid lines) of the numerically calculated (circles) wall-induced electrical lift force, $F_w$, for 5 µm (left) and 15 µm-diameter (right) spheres in a rectangular microchannel.

**Derivation of Eq. 8 in the main text for the lower limit in electric field for particle separation**

The transverse migration distance of a particle, $y$, due to the electrical lift in the main-branch of the bifurcating microchannel, is given by

$$y = L_{m-b} \frac{U_w}{U_{EK}} = L_{m-b} \frac{\varepsilon a E^2 \left( \frac{a}{h} \right)^4}{32 \eta \left( \frac{h}{a} \right)} = L_{m-b} \frac{a}{32 \left( \frac{h}{a} \right)} \left( \frac{a}{h} \right)^4 \frac{E}{\zeta_p - \zeta_w}$$

(S-3)

For particles that are focused to a stream along the centerline with one half of the channel width, $y = w/4$. Thus, Eq. S-3 is rewritten as
\[ E_{lower} = \frac{8}{L_{m-b}} \left( \frac{w}{a} \right) \left( \frac{h}{a} \right)^{4} (\zeta_{p} - \zeta_{w}) \] (S-4)

If the particle center-wall distance, \( h = w/8 \), which is one half of the overall migration distance for half focused particles (i.e., \( y = w/4 \)), is used to estimate the operating electric field, Eq. S-4 can be specified as Eq. 8 in the main text for the lower limit in electric field in the main-branch.

**Derivation of Eq. 9 in the main text for the upper limit in electric field for particle separation**

Particles that are deflected from the wall to the centerline of the side-branch experiences a transverse migration distance of one half of the channel width, i.e., \( y = w/2 \). Thus, Eq. S-3 can be rearranged to obtain the upper limit in electric field in the side-branch, \( E_{s-b}^{upper} \),

\[ E_{s-b}^{upper} = \frac{16}{L_{s-b}} \left( \frac{w}{a} \right) \left( \frac{h}{a} \right)^{4} (\zeta_{p} - \zeta_{w}) \] (S-5)

We then use the particle center-wall distance, i.e., \( h = w/4 \), which is one half of the overall migration distance for fully focused particles (i.e., \( y = w/2 \)), to estimate the electric field in Eq. S-5, leading to

\[ E_{s-b}^{upper} = \frac{1}{16L_{s-b}} \left( \frac{w}{a} \right)^{5} (\zeta_{p} - \zeta_{w}) \] (S-6)

Considering the electric field in the side-branch is one half of that in the main-branch for our current channel design, the value obtained from Eq. S-6 need to be doubled to estimate the electric field in the main-branch, i.e., \( E_{upper} \) in Eq. 9 of the main text.

**Superimposed image for the electrokinetic separation of 5 µm and 10 µm particles**
FIG. S-2 shows a top-view image of the separation of 5 µm and 10 µm-diameter spherical polystyrene particles at the expansion of one side-branch in a bifurcating microchannel. The inlet DC voltage is 1000 V, which is the same condition as that for the separation of 5 µm and 15 µm particles in FIG. 4C of the main text.

FIG. S-2. Top-view image of the electrokinetic separation of 5 µm and 10 µm-diameter spherical particles at the expansion of one side-branch in a bifurcating microchannel.

Analysis of the effect of particle shape on electrical lift force

The electrical lift, \( F_w \), in Eq. S-2 is also a function of particle shape (and hence potentially particle stiffness due to the shape changes of deformable particles in response to electric field). This hypothesis is based on the observation of dissimilar electric field distribution around a two-dimensional circular particle \((a = 5 \, \mu\text{m})\) of equal surface area in FIG. S-3A as compared to that around a two-dimensional elliptical particle \((a = 10/3 \, \mu\text{m} \text{ and } b = 15 \, \mu\text{m})\) in FIG. S-3B. It is further verified by comparing \( F_w \) on these two particles in a 100 µm-wide straight microchannel with an identical particle center-wall distance, \( h = 10 \, \mu\text{m} \), under a 40 kV/m DC electric field. The approach described in the preceding section via the surface integral analysis of the Maxwell stress tensor is used for the force calculation. As see from FIG. S-3C, \( F_w \) on the circular particle is always larger than that on the elliptical particle and nearly 5 times the latter at small \( h \) values.
Note that this relationship also remains valid for the two particles even with the same particle edge-wall distances, i.e., \((h-a)\). We believe that the difference in \(F_w\) between a spherical particle and a spheroidal particle of equal volume in a three-dimensional microchannel should be even larger than that shown in FIG. S-3C. Moreover, the sphere experiences a smaller drag force than the spheroid because the former has a smaller surface area. Therefore, \(F_w\) should deflect the spherical particle away from the wall at a higher rate than the spheroidal particle.

**FIG. S-3.** Comparison of the electric field lines and contours (the darker color, the larger magnitude) around a circular particle (A, \(a = 5 \mu m\)) and an elliptical particle (B, \(a = 10/3 \mu m\) and \(b = 7.5 \mu m\)) when the two particles travel electrophoretically through a straight rectangular microchannel with an identical \(h = 10 \mu m\) particle center-wall distance. The plot in (C) compares the wall-induced electric lift force, \(F_w\), on the two-dimensional circular and elliptical particles vs. the particle center-wall distance, \(h\).

**REFERENCES**


